Combining Adaptive Coding and Modulation with Hierarchical Modulation in Satcom Systems

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Abstract

We investigate the design of a broadcast system in order to maximise the throughput. This task is usually challenging due to the channel variability. Forty years ago, Cover introduced and compared two schemes: time sharing and superposition coding. Even if the second scheme was proved to be optimal for some channels, modern satellite communications systems such as DVB-SH and DVB-S2 mainly rely on time sharing strategy to optimize the throughput. They consider hierarchical modulation, a practical implementation of superposition coding, but only for unequal error protection or backward compatibility purposes. We propose in this article to combine time sharing and hierarchical modulation together and show how this scheme can improve the performance in terms of available rate. We introduce the hierarchical 16-APSK to boost the performance of the DVB-S2 standard. We also evaluate various strategies to group the receivers in pairs when using hierarchical modulation. Finally, we show in a realistic use case based on DVB-S2 that the combined scheme can provide throughput gains greater than 10% compared to the best time sharing strategy.

Index Terms

Broadcast Channel, Hierarchical Modulation, Digital Video Broadcasting.

I. INTRODUCTION

In most broadcast applications, the signal-to-noise ratio (SNR) experienced by each receiver can be quite different. For instance, in satellite communications the channel quality decreases with the presence of clouds in Ku or Ka

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band, or with shadowing effects of the environment in lower bands. The first solution for broadcasting is to design the system for the worst-case reception, but this leads to poor performance as many receivers do not exploit their full potential. Then, two other schemes have been proposed in [1] and [2]: time division multiplexing with variable coding and modulation, and superposition coding. Time division multiplexing, or time sharing, allocates to each user a fraction of time where it can use the full channel with any modulation and error protection level. This functionality, called variable coding and modulation (VCM) [3], is the most used in practice in standards today. If a return channel is available, VCM may be combined with adaptive coding and modulation (ACM) to optimize the transmission parameters [3]. In superposition coding, the available energy is shared to several service flows which are sent simultaneously in the same band. This scheme was introduced by Cover in [1] in order to improve the transmission rate from a single source to several receivers. When communicating with two receivers, the principle is to superimpose information for the user with the best SNR. This superposition can be done directly at the forward error correction (FEC) level or at the modulation level as shown in Figure 1 with a 16-QAM modulation. Hierarchical modulation is a practical implementation of superposition coding. Although hierarchical modulation has been introduced to improve the throughput, it is currently often used to provide unequal protection. The idea is to merge two different streams at the modulation level. The high priority (HP) stream is used to select the quadrant, and the low priority (LP) stream selects the position inside the quadrant. The HP stream is dedicated to users with poor channel quality, unlike the LP stream which requires a large SNR to be decoded. In [4], SVC encoded video [5] is protected using hierarchical modulation. The base layer of the video is transmitted in the HP stream, while the enhanced layer is carried by the LP stream. Another usage is backward compatibility [6], [7]. The DVB-S2 standard [3], [8] is called upon to replace DVB-S, but many DVB-S receivers are already installed. Thus the hierarchical modulation helps to the migration by allowing the DVB-S receivers to operate. In [9], the authors propose to provide local content with hierarchical modulation. The principle is to carry locale information that is of interest to a particular geographic area in the LP stream, while the HP stream transmits global content. Finally, other works improve the performance of relay communication system [10] or OFDMA-based networks [11].

Our work focuses on using hierarchical modulation in modern broadcast systems to increase the transmission rate. For instance, even if the LDPC codes of DVB-S2 approach Shannon limit for the AWGN channel with one receiver [12], the throughput can be greatly increased for the broadcast case. Indeed, Cover presents the set of achievable rates for the Gaussian broadcast channel with two receivers in [1]. This set clearly dominates the time sharing achievable rate. This article investigates the performance, in terms of throughput, of a satellite broadcast system where time sharing and hierarchical modulation are combined. We show on an example modeling a satellite broadcast area that the gain can be significant. The way of grouping receivers in pairs is also investigated as it greatly affects the performance.

The paper is organized as follows. Section II presents the hierarchical modulation. We introduce the hierarchical 16-APSK in order to boost the performance of the DVB-S2 standard. In Section III, the achievable rates are computed for the time sharing used alone or combined with hierarchical modulation. We study on a use case the performance of each scheme in Section IV. We also propose a grouping strategy when using hierarchical modulation

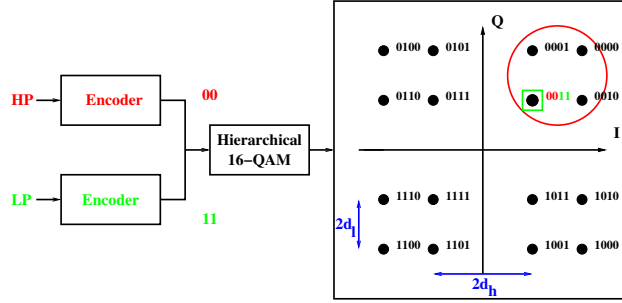and discuss its performance. Finally, Section V concludes the paper by summarizing the results.

Fig. 1: Hierarchical modulation using a 16-QAM

II. HIERARCHICAL MODULATION

This part introduces the hierarchical 16-QAM and 16-APSK. First, the hierarchical 16-QAM, considered in the DVB-SH standard [13],[14], is presented. It gives us some insight on how to introduce the hierarchical 16-APSK. Then the hierarchical 16-APSK is presented in order to improve the performance of DVB-S2. As mentioned before, hierarchical modulations merge several streams in a same symbol. The available energy is shared between each stream. In our study, two streams are considered. When hierarchical modulation is used to unequal protection purpose, these flows are called HP and LP streams. However, unequal protection is not the goal of our work, so we will now refer to high energy (HE) and low energy (LE) streams for the stream containing the most and the least energy respectively.

As each stream does not use the same energy, hierarchical modulations are based on non-uniform constellations where the symbols are not uniformly distributed in the space. The geometry of non-uniform constellations is described using the constellation parameter(s).

A. Hierarchical 16-QAM

The constellation parameter α is defined by d_h/d_l, where $2d_h$ is the minimum distance between two constellation points carrying different HE bits and $2d_l$ is the minimum distance between any constellation point (see Figure 1). Typically, we have $\alpha \geq 1$, where $\alpha = 1$ corresponds to the uniform 16-QAM, but it is also possible to have $\alpha \leq 1$ [15]. When α grows, the constellation points in each quadrant become farer to the I and Q axes. Thus it is easier to decode the HE stream. However, in the same quadrant, the points get closer and the LE stream requires a better channel quality to be decoded.

The hierarchical 16-QAM is the superposition of two QPSK modulations, one with parameter $d_h + d_l$ (carrying the HE stream) and the other with parameter d_l (carrying the LE stream). The energy ratio between the two streams is

$$\frac{E_{he}}{E_{le}} = (1 + \alpha)^2, \tag{1}$$

where E_{he} and E_{le} correspond to the energy allocated to the HE and LE streams respectively [9]. The DVB-SH standard recommends two values for α: 2 and 4. In fact, it also considers $\alpha = 1$ but only for the VCM mode. The values 2 and 4 are defined in order to provide unequal protection. From an energy point of view, this amounts to give 90% ($\alpha = 2$) or 96% ($\alpha = 4$) of the available energy to the HE stream. In [16], the authors improve the overall throughput of a simple broadcast channel by adding $\alpha = 1$, $\alpha = 0.8$ and $\alpha = 0.5$. This provides a better repartion of the energy: the HE stream contains 80%, 76% and 69% of the total power.

B. Hierarchical 16-APSK

The DVB-S2 standard also introduces hierarchical modulation with the hierarchical 8-PSK. The constellation parameter θ_{PSK}, which is the half angle between two points in one quadrant, is defined by the service operator according to the desired performance. However, this modulation does not offer a good spectrum efficiency. As the 16-APSK is already defined in DVB-S2, we propose the hierarchical 16-APSK, shown in Figure 2, in order to boost the performance of the system. Two parameters impact on the constellation geometry: $\gamma = R_2/R_1$ and θ.
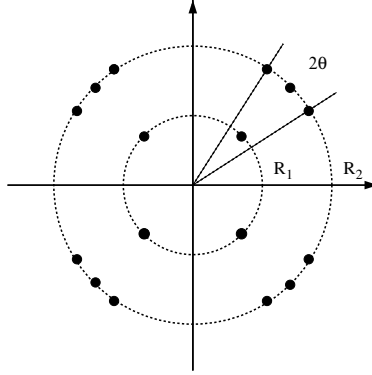

Fig. 2: 16-APSK modulation

The hierarchical 16-APSK is not a new concept. For instance, this modulation is presented to upgrade an existing digital broadcast system in [17]. However, the design of the modulation is not addressed. In [18], the authors investigate the design of APSK modulations for satellite broadband communication, but the hierarchical case is not treated. In our study, we use an energy argument to choose the parameters γ and θ. The hierarchical modulation shares the available energy between the HE and LE streams. We consider the energy of the HE stream. It is given by the energy of a QPSK modulation, where the constellation points are located at the barycenter of the four points in each quadrant. Using the polar coordinates, the barycenter in the upper right quadrant is

$$\frac{R_1 e^{i\pi/4} + R_2 e^{i\pi/4} + R_2 e^{i(\pi/4+\theta)} + R_2 e^{i(\pi/4-\theta)}}{4} = e^{i\pi/4} \frac{R_1 + R_2 + 2R_2 \cos(\theta)}{4}. \tag{2}$$

Moreover, the symbol energy is expressed as

$$\begin{aligned} E_s &= \frac{4R_1^2 + 12R_2^2}{16} \\ &= \frac{1 + 3\gamma^2}{4} R_1^2. \end{aligned} \tag{3}$$

Then combining (2) and (3), the distance of the barycenter to the origin is

$$
\begin{aligned}
d_B &= \frac{R_1 + R_2 + 2R_2 \cos(\theta)}{4} \\
&= \frac{1 + \gamma(1 + 2\cos(\theta))}{4} R_1 \\
&= \frac{1 + \gamma(1 + 2\cos(\theta))}{4} \frac{2\sqrt{E_s}}{\sqrt{1 + 3\gamma^2}}.
\end{aligned}
\tag{4}
$$

Finally, the energy of the HE stream is given by

$$
\begin{aligned}
E_{he} &= E_{qpsk} \\
&= d_B^2 \\
&= \underbrace{\frac{(1 + \gamma(1 + 2\cos\theta))^2}{4(1 + 3\gamma^2)}}_{\rho_{he}} E_s.
\end{aligned}
\tag{5}
$$

Equation (5) introduces ρ_{he} the ratio between the energy of the HE stream E_{he} and the symbol energy E_s. We are now interested to determine the set of (γ, θ) pairs solution of (6), where $\rho_{he} \geq 0.5$ is known, $\gamma \geq 1$ and $\theta \geq 0$.

$$
\rho_{he} = \frac{(1 + \gamma(1 + 2\cos\theta))^2}{4(1 + 3\gamma^2)}
\tag{6}
$$

The resolution of (6) is given in Appendix A. The solution set is

$$
\mathcal{S}_{\rho_{he}} = \left\{ (\gamma, \arccos(f(\gamma, \rho_{he}))) \, | \, 1 \leq \gamma \leq \gamma_{lim} \right\},
\tag{7}
$$

where

$$
f(\gamma, \rho_{he}) = \frac{1}{2} \left(\frac{\sqrt{4\rho_{he}(1 + 3\gamma^2)} - 1}{\gamma} - 1 \right)
\tag{8}
$$

and

$$
\gamma_{lim} = \begin{cases} +\infty, & \text{if } \rho_{he} \leq 0.75 \\ \frac{3 + 4\sqrt{3\rho_{he}(1 - \rho_{he})}}{3(4\rho_{he} - 3)}, & \text{if } \rho_{he} > 0.75. \end{cases}
\tag{9}
$$

Appendix A also presents two examples of the $\mathcal{S}_{\rho_{he}}$ set. When ρ_{he} increases, the points in one quadrant tend to get closer. It implies the HE stream is easier to decode, but on the other hand the LE stream requires a better SNR to be decoded. As for the hierarchical 16-QAM, several values of ρ_{he} have to be selected. The results presented in Section IV show that choosing $\rho_{he} = 0.75, 0.8, 0.85$ and 0.9 allows to have good performance. However, once the parameter ρ_{he} is set, we still have to decide which (γ, θ) couple to keep in the $\mathcal{S}_{\rho_{he}}$ set. We keep one (γ, θ) pair per ρ_{he} value. This pair minimizes the average decoding threshold for the HE stream over all the coding rates in the DVB-S2 standard. Appendix B gives all the decoding thresholds and explains their computation.

III. TIME SHARING AND HIERARCHICAL MODULATION ACHIEVABLE RATES

This part introduces the rates achievable by the two following schemes: time sharing with or without hierarchical modulation, referred as *hierarchical modulation* and *classical* time sharing respectively. We first consider the case of one source communicating with two receivers. Then we study the general case with n receivers.

A. Achievable rates: case with two receivers

1) Classical time sharing: We consider one source communicating with two receivers, each one with a particular SNR. Given this SNR, we suppose receiver i ($i = 1, 2$) has a rate R_i, which corresponds to the best rate it can afford. This rate depends on the modulations and coding rates available in the system. For the DVB-S2 standard, readers can refer to [8]. With a time sharing strategy, the average rate results from the fraction of time t_i allocated to each receiver. In our study, we are interested to offer the *same rate* to everyone, but our work can be easily adapted to another rate policy. To offer the same rate to the two users, we need to solve

$$\begin{cases} t_1 R_1 = t_2 R_2 \\ t_1 + t_2 = 1. \end{cases} \tag{10}$$

By solving (10), the fraction of time allocated to each user is

$$\begin{cases} t_1 = \frac{R_2}{R_1 + R_2} \\ t_2 = \frac{R_1}{R_1 + R_2}. \end{cases} \tag{11}$$

The constraint $t_1 + t_2 = 1$ is verified and we remark that increasing R_i reduces t_i, which is a consequence of our rate policy. Finally, the rate offered to each receiver is

$$R_{ts} = \frac{R_1 R_2}{R_1 + R_2}. \tag{12}$$

2) Hierarchical modulation time sharing: The first step is to compute the rates offered by all the possible modulations, including hierarchical ones. When the hierarchical modulation is used, we suppose *the receiver experiencing the best SNR decodes the LE stream*. Thus we obtain a set of operating points. Moreover, when two sets of rates (R_1, R_2) and $\left(\tilde{R}_1, \tilde{R}_2 \right)$ are available, the time sharing strategy allows any rate pair

$$\left(\tau R_1 + (1 - \tau)\tilde{R}_1, \tau R_2 + (1 - \tau)\tilde{R}_2 \right), \tag{13}$$

where $0 \leq \tau \leq 1$ is the fraction of time allocated to (R_1, R_2). The achievable rates set finally corresponds to the convex hull of all the operating points. As we are interested to offer the *same rate* to the users, we calculate the intersection of the convex hull with the curve $y = x$. We note R_{hm} and R_{ts} the rates offered to both receivers by the hierarchical modulation and classical time sharing strategy respectively.

Figure 3 presents one example of achievable rates set where one receiver experiences a SNR of 7 dB and the other 10 dB. We also represent R_{hm} and R_{ts} in order to visualize the gain. For the classical time sharing strategy, the rates obtained in Figure 3 come from the 8-PSK 2/3 for the user with a SNR of 7 dB and the 16-APSK 3/4 for the other user. As said before, it is the best that each receiver can afford. For the hierarchical modulation, the operating points are computed using the standard [8] or Appendix B. Remark that the hierarchical 16-APSK gives better results than the hierarchical 8-PSK. Moreover, when ρ_{he} increases, the rate of the user with the worst SNR increases, while the rate of the other user decreases. This can be explained as more energy is dedicated to the HE stream and then its performance improves. On the other hand, the LE stream is allocated less energy and its
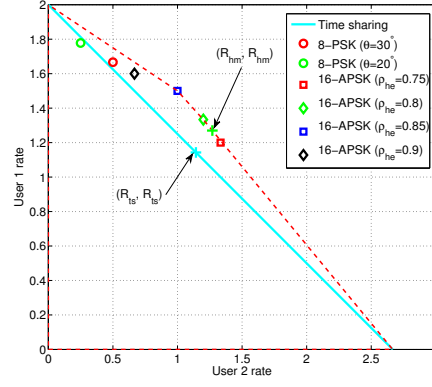
Fig. 3: Achievable rates set: $SNR_1 = 7$ dB - $SNR_2 = 10$ dB

performance decreases. Finally, the interest of using hierarchical modulation is obvious as the gain between R_{hm} and R_{ts} is about 11%.

We now use the same method to evaluate the gain between R_{hm} and R_{ts} for all (SNR_1, SNR_2) pairs where 4 dB $\leq SNR_i \leq 12$ dB, $i = 1, 2$. Figure 4 presents the results. Note that the gain provided by the hierarchical modulation is significative in several cases and can achieve up to 20%. In general, the gains are more important when the SNR difference between the two receivers is large. This observation will be used in the next part when we will group a set of users in pairs.
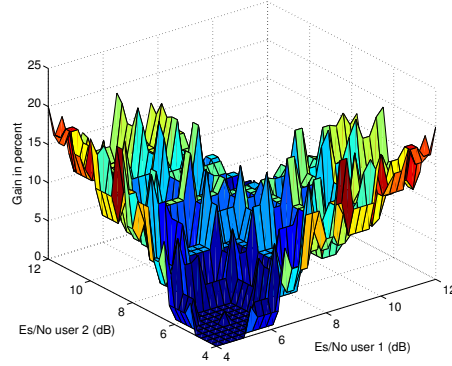
Fig. 4: Rate gain

B. Achievable rates: case with n receivers

1) Classical time sharing: We now consider a broadcast system with n receivers. We suppose receiver i has a rate R_i, which corresponds to the best rate it can afford as mentioned above. With our rate policy, (10) becomes

$$\begin{cases} \forall i, j, \ t_i R_i = t_j R_j \\ \sum_i t_i = 1. \end{cases} \tag{14}$$

The resolution of (14) leads to a fraction of time allocated to user i of

$$t_i = \frac{\prod_{k \neq i} R_k}{\sum_{j=1}^n \left(\prod_{k \neq j} R_k \right)}. \tag{15}$$

With this time allocation, the average rate offered to each receiver is

$$R_{ts} = \frac{\prod_k R_k}{\sum_{j=1}^n \left(\prod_{k \neq j} R_k \right)} = \left(\sum_{j=1}^n \frac{1}{R_j} \right)^{-1}. \tag{16}$$

2) Hierarchical modulation with time sharing: For the case with n receivers, the first step is to group the users in pairs in order to use hierarchical modulation. A lot of possibilities are available and the next section presents a grouping strategy which generally obtains good results. Once the pairs have been chosen, for each pair we compute the achievable rate as described previously. Finally, we need to equalize the rate between each user. This is done by time sharing using (15). For instance, consider a DVB-S2 system where a user u_1 with a SNR of 7 dB is in pair with a user u_2 with a SNR of 10 dB. The rate for each receiver is obtained using the hierarchical 16-APSK ($\rho_{he} = 0.8$) a fraction of time a_1 and the 16-APSK a fraction of time a_2 as shown in Figure 3. When equalizing the rates between all the users, (15) gives the same fraction of time t to users u_1 and u_2. Then, the pair of users has to share a global fraction of time $2t$. It follows that the broadcast system allocates to u_1 and u_2 the hierarchical 16-APSK ($\rho_{he} = 0.8$) for a time proportion $2t \times a_1$ and the 16-APSK for $2t \times a_2$.

IV. APPLICATION TO BROADCAST CHANNEL

An important insight of the previous section is that the gain of hierarchical modulation depends on the way to group users in pairs. For instance, consider a set of four users u_1, u_2, u_3, u_4 with respective SNR 4, 4, 12 and 12 dB. Then, according to Figure 4, the choice of pairs (u_1, u_2) and (u_3, u_4) leads to no gain while the choice (u_1, u_3) and (u_2, u_4) allows a gain of about 20%. In this section, we first present different grouping strategies of a set of users when the hierarchical modulation time sharing is considered. Then we introduce a broadcast channel where the performance of hierarchical modulation and classical time sharing are evaluated. The impact of the grouping strategy is also discussed.

A. Grouping strategy

We consider a set of receivers, where the distribution of the SNR values is known. The possible SNR values are SNR_i with $1 \leq i \leq m$ and for all $i \leq j$, $SNR_i \leq SNR_j$. Moreover, exactly n_i receivers experience a channel quality of SNR_i. We also define $\sum_{i=1}^m n_i = 2N$ the total number of receivers and $\Delta_{i,j} = |SNR_i - SNR_j|$.

Definition 1: For any grouping, the average SNR difference per receivers in pairs is defined as

$$\Delta = \frac{1}{N} \sum_{k=1}^{N} \Delta_{i_k, j_k}, \tag{17}$$

where the (i_k, j_k) couple represents a pair of receivers.

When communicating with two receivers, we have already mentioned that the gain is more important when the SNR difference between the two users is large. From this observation, we are looking to group the users in pairs in order to maximise the average SNR difference. The following theorem presents a strategy to compute this maximum.

Theorem 1: From any set of receivers, the iterative procedure that picks the two receivers with the largest SNR difference, group them and repeat this operation allows to reach the maximum average SNR difference.

Proof: First, we compute an upper bound for the average SNR difference. In (17), for all k, we have (assuming $i_k \leq j_k$)

$$\Delta_{i_k, j_k} = \sum_{m=i_k}^{j_k - 1} \Delta_{m, m+1}. \tag{18}$$

Thus, Δ can be expressed in the following form

$$\Delta = \frac{1}{N} \sum_{i=1}^{m-1} a_i \Delta_{i, i+1}, \tag{19}$$

where $a_i \in \mathbb{N}$ for all i. We now search to bound a_i. The term $\Delta_{i, i+1}$ in (19) only appears when we group a user with a SNR less or equal to SNR_i and a user with a SNR greater or equal to SNR_{i+1}. There is exactly $\sum_{k=1}^{i} n_k$ receivers with $SNR \leq SNR_i$ and $\sum_{k=i+1}^{m} n_k$ receivers with $SNR \geq SNR_{i+1}$, so a_i is bounded by

$$a_i \leq \min(\sum_{k=1}^{i} n_k, \sum_{k=i+1}^{m} n_k). \tag{20}$$

We now prove the proposed scheme reaches this bound, i.e., $a_i = \min(\sum_{k=1}^{i} n_k, \sum_{k=i+1}^{m} n_k)$. Let L be the greatest integer such as $\sum_{i=1}^{L} n_i \leq N$. The strategy insures that all the receivers with a SNR less or equal to SNR_{L+1} are grouped with a receiver whose SNR is greater or equal to SNR_{L+1}. Thus, in the computation of the average SNR difference, we verify that

- $\Delta_{1,2}$ appears $n_1 = \min(\sum_{k=1}^{1} n_k, \sum_{k=2}^{m} n_k)$ times.
- $\Delta_{2,3}$ appears $n_1 + n_2 = \min(\sum_{k=1}^{2} n_k, \sum_{k=3}^{m} n_k)$ times.
- ...
- $\Delta_{L,L+1}$ appears $n_1 + n_2 + ... + n_L = \min(\sum_{k=1}^{L} n_k, \sum_{k=L+1}^{m} n_k)$ times.

The equality also holds in (20) for the terms $\Delta_{i, i+1}$ with $i \geq L+1$. Thus, our strategy allows to reach the previous bound, which is in fact the maximum average SNR difference. ■

However, depending on the receivers configuration, other schemes allow to reach the maximum average SNR difference. For instance, consider the case with four receivers where the SNR values are SNR_1 (user 1), $SNR_2 = SNR_1 + 1$ dB (user 2), $SNR_3 = SNR_1 + 2$ dB (user 3) and $SNR_4 = SNR_1 + 3$ dB (user 4). The previous

strategy leads to group user 1 with user 4, and user 2 with user 3. But it is also possible to group user 1 with user 3, and user 2 with user 4. In both cases, the average SNR difference is 2 dB.

Moreover, to highlight the impact of the grouping strategy, we propose to compare four grouping schemes:

- Strategy A: the scheme described in the previous theorem.
- Strategy B: we compute the maximum average SNR difference Δ_{max} and use it to group the receivers with a SNR difference as close as possible to Δ_{max}. This strategy usually allows to have an average SNR difference close to Δ_{max}, but compared to strategy A, the variance of the SNR difference by pairs is much smaller.
- Strategy C: the receivers are grouped randomly.
- Strategy D: we group the receivers with the closest SNR.

B. DVB-S2 channel model

To evaluate the effective potential of our proposal for real systems, we present a model to estimate the SNR distribution of the receivers. For that, we considered the set of receivers located in a given spot beam of a geostationnary satellite broadcasting in the Ka band. The model takes into account two main sources of attenuation: the relative location of the terminal with respect to the center of (beam) coverage and the weather. Concerning the attenuation due to the location, the principle is to set the SNR at the center of the spot beam SNR_{max} and use the radiation pattern of a parabolic antenna to model the attenuation. An approximation of the radiation pattern is

$$G(\theta) = G_{max} \left(2 \frac{J_1 \left(\sin(\theta) \frac{\pi D}{\lambda} \right)}{\sin(\theta) \frac{\pi D}{\lambda}} \right)^2, \tag{21}$$

where J_1 is the first order Bessel function, D is the antenna diameter and $\lambda = c/f$ is the wavelength [19]. In our simulations, we use $D = 1.5$ m and $f = 20$ GHz. Moreover we consider a typical multispot system where the edge of each spot beam is 4 dB below the center of coverage. Assuming a uniform repartition of the population, the proportion of the receivers experiencing an attenuation between two given values is the ratio of the ring area over the disk shown in Figure 5. The ring area is computed knowing the satellite is geostationary and using (21).
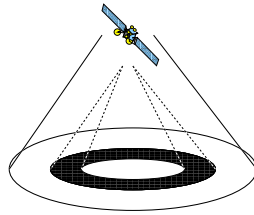

Fig. 5: Satellite broadcasting area

Figure 6, provided by the CNES, presents the attenuation distribution in the broadcasting satellite service (BSS) band. More precisely, it is a temporal distribution for a given location in Toulouse, France. In our work, we assume the SNR distribution for the receivers in the beam coverage at a given time is equivalent to the temporal distribution at a given location.
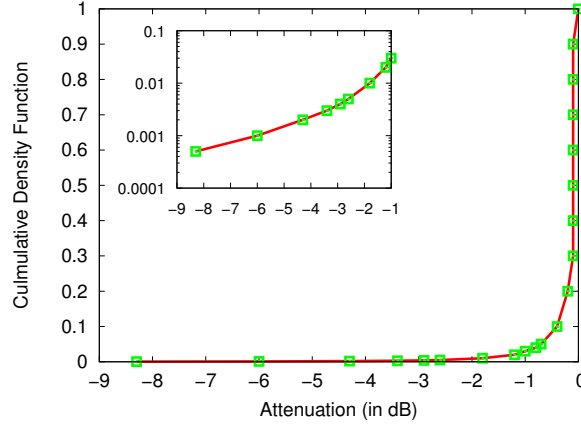
Fig. 6: Attenuation distribution (due to weather)

Finally, our model combines the two kinds of attenuation to estimate the SNR distribution. From a set of receivers, we first compute the attenuation due to the location. Then, for each receiver we draw, according to the previous distribution, the attenuation caused by the weather.

C. Results

Two scenarios are considered. In the first one, all the terminals have the same capacities (homogeneous case). In the second one, we consider two subsets of receivers, one with personal terminals and the other with professional terminals (heterogeneous case). Professional terminals forward the service, and not the signal, to some receivers in a local area network. We suppose the professional terminals experience 5 dB better than the personal terminals. They allow to increase the SNR diversity which is interesting when using the hierarchical modulation time sharing. Figure 7 shows a broadcast channel with two kinds of terminals (large antennas represent professional terminals).
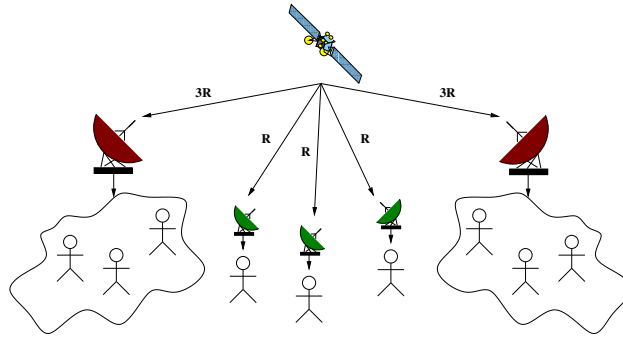

Fig. 7: Broadcast channel with two kinds of terminals

1) Homogeneous terminals: Figure 8 presents the results for a broadcasting area with 500 receivers. For each simulation, the SNR value of each user is drawn according to the distribution presented above. For one configuration

(i.e., the parameter SNR_{max} is set), we present the average, minimum and maximum gains over 100 simulations for the four grouping strategies.
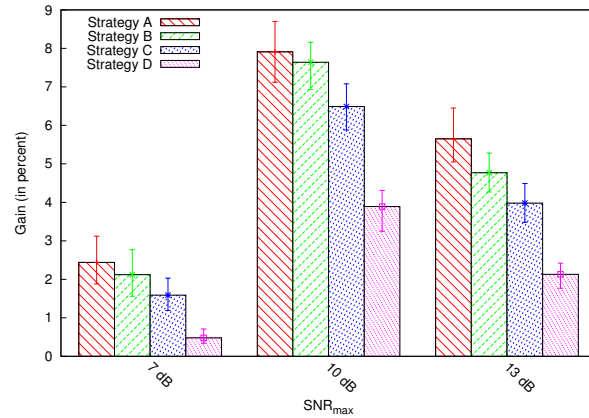

Fig. 8: Results for the homogeneous case with 500 receivers

First of all, the hierarchical modulation time sharing outperforms the classical time sharing scheme whatever the grouping strategy used. In fact, the hierarchical modulation adds some new operating points and thus can only improve the performance.

Then, for each configuration, strategies A and B give the best results with a slight advantage for strategy A, which obtains more than 9% of gain for $SNR_{max} = 10$ dB. This is consistent with the results presented in Figure 4, where the best gains are obtained when the SNR difference between the two receivers is large. Moreover, strategy D, which minimises the SNR difference, appears to be the worst scheme. The results also point out that the random strategy performs well. Thus the hierarchical modulation time sharing combined with a good grouping strategy allows to obtain intermediate gains between strategies A and D. In addition, strategies A, B and C do not require intensive computation to group the receivers and this can be done in real time which is interesting for satellite standards.

We remark the better results are obtained when $SNR_{max} = 10$ dB. In fact, the hierarchical modulation time sharing is useful only in a SNR interval. Figure 9 presents the gain of the strategy A according to SNR_{max} for a large range of SNR_{max} values. For low SNR values, the LE stream is often not able to decode any coding rate. This explains why there is no gain for low SNR values. An idea to tackle this phenomenon is to allocate more energy to the LE stream, but in that case, the performance of the HE stream is too deteriorated. For large SNR values, the classical time sharing uses the largest coding rate possible. For instance, consider two receivers with a SNR greater than 13.13 dB which corresponds to the decoding threshold of the 9/10 16-APSK [8]. The classical time sharing strategy allocates the same fraction of time $t = 0.5$ to both receivers. For the hierarchical modulation time sharing, one of the receiver decodes the HE stream, and the other the LE stream. In the best case, each stream can decode the coding rate 9/10. Each receiver use the channel all the time but the HE and LE streams

only carry two bits. Then the hierarchical modulation time sharing can not outperform the classical scheme here. This example illustrates why the hierarchical modulation time sharing does not increase the performance for large value of SNR_{max}. A solution should be to use a higher order modulation, for instance the 32-APSK.
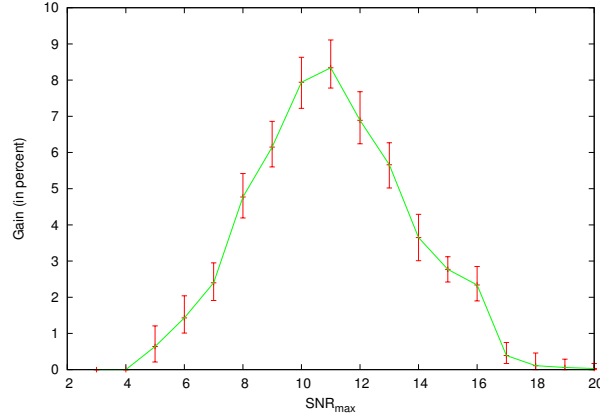

Fig. 9: Gain vs SNR_{max} (Strategy A)

2) Heterogeneous terminals: We investigate the scenario where personal and professional terminals are used simultaneously. We consider that only one receiver is served by a personal terminal whereas there are several users behind a professional terminal (see Figure 7). As mentioned before, the professional terminals experience 5 dB better than the personal terminals. It is also important to notice that the rate dedicated to one professional terminal is proportional to the number of receivers served by this terminal. The aim is always to provide the same rate for each user.

In this scenario, the performance depends on the proportion of receivers served by a professional terminal and SNR_{max}. Here SNR_{max} corresponds to the SNR experienced by personal terminals at the center of the spot beam (with clear sky condition). Figure 10 presents the gains according to the proportion of users served by a collective terminal. Here again, the simulations involve 500 receivers and we present the results over 100 simulations. First of all, the gains are most of the time better than in Figure 8. This is in accordance with Figure 4 as the presence of professional terminals increases the average SNR difference. Then, for a given SNR_{max}, the maximum gain takes place when 50% of the receivers are served by a professional terminal, which corresponds to the maximum possible SNR diversity. This result is consistent with the work presented in [11]. Finally, compared to Figure 8, the results are worse in two cases, when $SNR_{max} = 10$ or 13 dB and 90% of the receivers are served by a professional terminal. In these particular cases, we do not really increase the SNR diversity, but rather the average SNR. Then, the performance when $SNR_{max} = 10$ dB and 90% of the receivers are served by professional terminals is more similar to the performance observed in Figure 10 when $SNR_{max} = 15$ dB (we assume that a professional terminal experiences 5 dB better than a personal terminal).
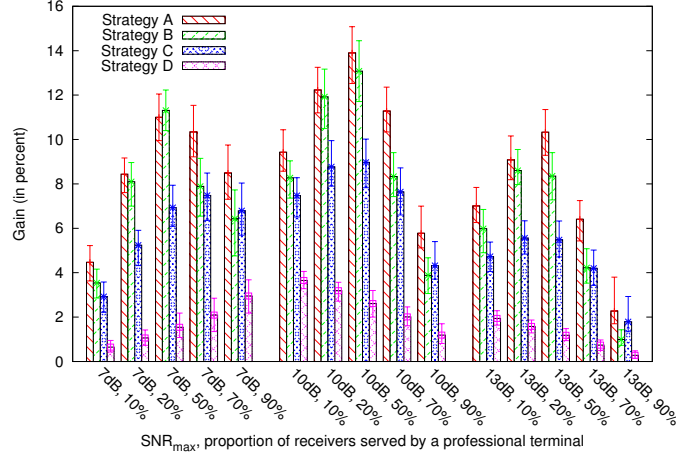
Fig. 10: Results for the heterogeneous case with 500 receivers

V. CONCLUSION

In this paper, we use time sharing and hierarchical modulation together to increase the throughput of a broadcast channel. We first propose the hierarchical 16-APSK to generalise the use of hierarchical modulation for the DVB-S2 standard. To the best of our knowledge, the hierarchical 16-APSK has not been extensively studied. Here we choose the constellation parameters according to an energy argument. Then we present how to compute the achievable rates for our scheme. We introduce several strategies to group the users in pairs. We propose two scenarios including homogeneous and non-homogeneous terminals and show that a gain of roughly 15% can be achieved (in the best case) by the strategy grouping the receivers with the greatest SNR difference.

In this paper, we study the case where all the receivers get the same rate. Future works will extend our work to any rate policy. We also expect to study the gain of using hierarchical modulation in other standards (e.g., terrestrial standards).

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APPENDIX A
RESOLUTION OF THE ENERGY EQUATION

Consider the equation

$$\rho_{he} = \frac{(1 + \gamma(1 + 2\cos\theta))^2}{4(1 + 3\gamma^2)}, \tag{22}$$

where $\rho_{he} \geq 0.5$ is known, $\gamma \geq 1$ and $\theta \geq 0$. From (22), it follows

$$\begin{aligned}
\cos\theta &= \frac{1}{2}\left(\frac{\sqrt{4\rho_{he}(1 + 3\gamma^2)} - 1}{\gamma} - 1\right) \\
&= f(\gamma, \rho_{he}). \tag{23}
\end{aligned}$$

We are now looking when $-1 \leq f(\gamma, \rho_{he}) \leq 1$ in order to use the $\arccos$ function. The function $f(\gamma, \rho_{he})$ is an increasing function of γ when ρ_{he} is set. Moreover,

$$f(\gamma, \rho_{he}) \xrightarrow[\gamma \to +\infty]{} \frac{1}{2}(2\sqrt{3\rho_{he}} - 1). \tag{24}$$

The right term is an increasing function in ρ_{he} and equals 1 for $\rho_{he} = 0.75$. Thus, for all $\rho_{he} \leq 0.75$, the solution is

$$\mathcal{S}_{\rho_{he}} = \{(\gamma, \arccos(f(\gamma, \rho_{he}))) \,|\, \gamma \geq 1\}. \tag{25}$$

When $\rho_{he} > 0.75$, γ stay bounded. To determine the limit value γ_{lim}, we have to solve the equation,

$$\frac{1}{2}\left(\frac{\sqrt{4\rho_{he}(1 + 3\gamma^2)} - 1}{\gamma} - 1\right) = 1 \quad \Leftrightarrow \quad \sqrt{4\rho_{he}(1 + 3\gamma^2)} - 1 = 3\gamma$$

$$\Leftrightarrow \quad (12\rho_{he} - 9)\gamma^2 - 6\gamma + (4\rho_{he} - 1) = 0. \tag{26}$$

It is a quadratic equation with discriminant $\Delta = 192\rho_{he}(1 - \rho_{he})$. The solutions are

$$s_{1,2} = \frac{6 \pm \sqrt{192\rho_{he}(1 - \rho_{he})}}{2(12\rho_{he} - 9)}. \tag{27}$$

We keep the positive solution,

$$\gamma_{lim} = \frac{3 + 4\sqrt{3\rho_{he}(1 - \rho_{he})}}{3(4\rho_{he} - 3)}. \tag{28}$$

Finally, the solution of (22) for $\gamma > 0.75$ is,

$$\mathcal{S}_{\rho_{he}} = \{(\gamma, \arccos(f(\gamma, \rho_{he}))) \,|\, 1 \leq \gamma \leq \gamma_{lim}\}. \tag{29}$$

Figure 11 presents two examples of $\mathcal{S}_{\rho_{he}}$ with different values of ρ_{he}.

(a) $\rho_{he} = 0.8$

(b) $\rho_{he} = 0.9$
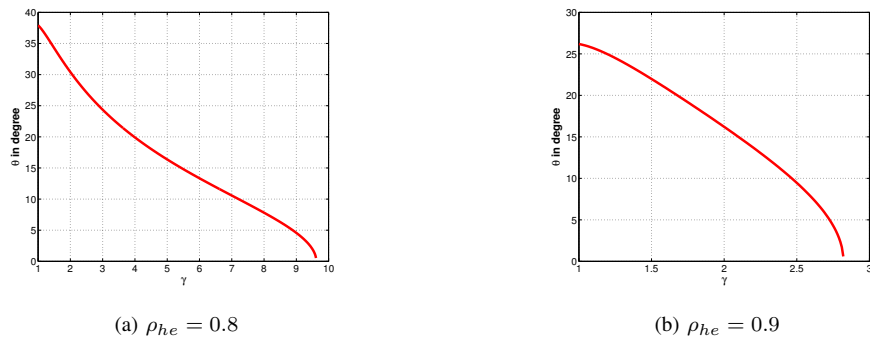
Fig. 11: Example of $\mathcal{S}_{\rho_{he}}$

APPENDIX B

HIERARCHICAL 16-APSK PERFORMANCE

We develop in this section the method used to choose the (γ, θ) pair for the hierarchical 16-APSK once the parameter ρ_{he} has been set. We decide to keep only one (γ, θ) pair per ρ_{he} value, as the simulations to obtain the performance are time consuming. For a given ρ_{he}, the decoding thresholds for all the coding rates in function of γ are estimated using the method described in [20]. This allows to obtain a fast approximation of all the decoding thresholds. For instance, Figure 12 presents the curves obtained for $\rho_{he} = 0.8$, where the crosses correspond to the minimum of each curve. Note that the mathematical resolution of (22) allows large value of γ. In our work, we decide to upper bound γ by $\min(5, \gamma_{lim})$.
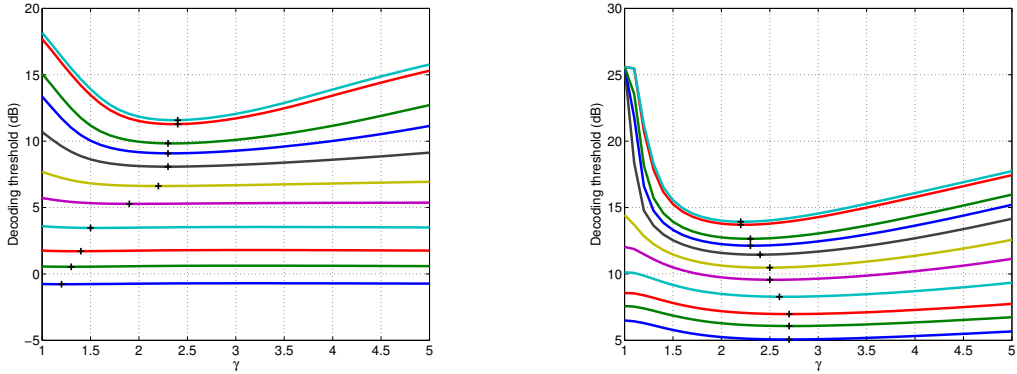
(a) Estimated HE stream decoding thresholds, $\rho_{he} = 0.8$ (b) Estimated LE stream decoding thresholds, $\rho_{he} = 0.8$

Fig. 12: Estimated performance of the hierarchical 16-APSK

Then we choose to adopt the (γ, θ) pair that minimizes the average decoding threshold for the HE stream over all the coding rates. Figure 12 shows that this solution does not penalize too much the LE stream. With the estimated performance, we pick the (γ, θ) pair according to the previous criteria. Table I present the adopted pairs.

TABLE I: Adopted (γ, θ) values

| ρ_{he}	0.75	0.8	0.85	0.9
γ	2.8	2.3	1.9	1.6
θ	31.5	28.4	25.1	20.9

Finally, the performance is evaluated with simulations using the Coded Modulation Library [21] that already implements the DVB-S2 LDPC. The LDPC codewords are 64 800 bits long (normal FEC frame) and the iterative decoding stops after 50 iterations if no valid codeword has been decoded. Moreover, in our simulations, we wait until 10 decoding failures before computing the BER. If the BER is less than 10^{-4}, then we stop the simulation.
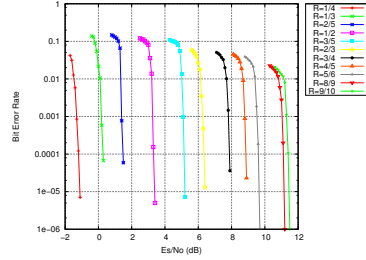
Our stopping criterion is less restrictive than in [3] (i.e, a Packet Error Rate of 10^{-7}) because simulations are time consuming. However, our simulations are sufficient to detect the waterfall region of the LDPC and then the performance of the code. Figure 13 presents all the BER curves.
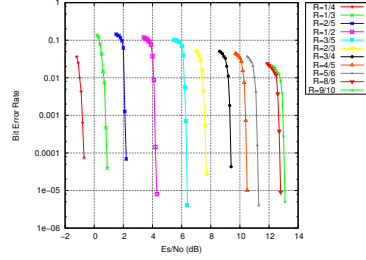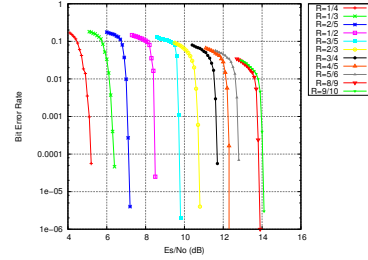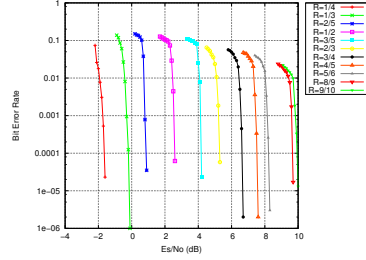

(a) HE stream performance, $\rho_{he} = 0.75$
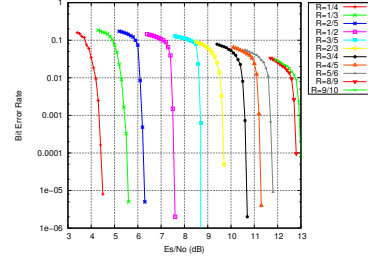
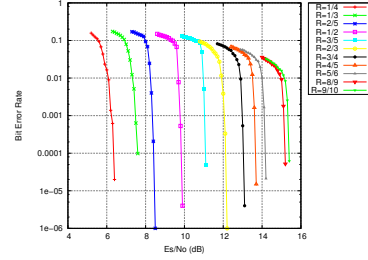(b) LE stream performance, $\rho_{he} = 0.75$

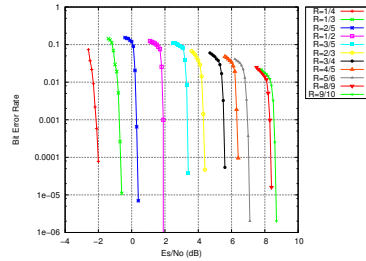(c) HE stream performance, $\rho_{he} = 0.8$

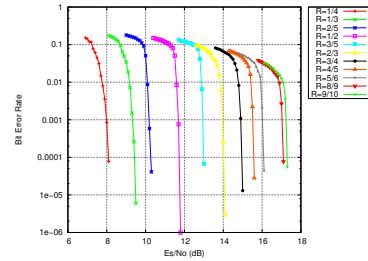(d) LE stream performance, $\rho_{he} = 0.8$

(e) HE stream performance, $\rho_{he} = 0.85$

(f) LE stream performance, $\rho_{he} = 0.85$

(g) HE stream performance, $\rho_{he} = 0.9$

(h) LE stream performance, $\rho_{he} = 0.90$

Fig. 13: Performance of the hierarchical 16-APSK

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